THE MOTION OF A LIQUID WITH VARIABLE RHEOLOGICAL CHARACTERISTICS IN A CIRCULAR CYLINDRICAL TUBE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 2, pp. 304–308, 1968

UDC 532.135+532.542

We present the results of experimental investigations and give the rheological equation of state for the media which have been studied. A general equation of mechanics has been derived, and we present a solution for the problem of the motion (in a circular cylindrical tube) of liquids with variable rheological characteristics.

We studied the rheological properties of polymer systems based on chlorosulfonated polyethylene and polystyrene, in which mineral pigments, cement, and similar additives served as fillers.

To carry out our tests, we devised a special thermostating insulation in which the studies were carried out over an extremely extensive range of strain rates under conditions of virtually uniform shear ($4.5 \cdot 10^{-4} \le \dot{e} \le 2.2 \cdot 10^4\ \mathrm{sec}^{-1}$). This yielded "flow curves" for solutions filled with chlorosulfonated polyethylene and polystryrene in several solvents with various fillers, as the temperature was varied.

The resulting functional curves for the tangential shear stress relative to the strain-rate gradient show (Fig. 1) that in virtually every case the rheological behavior of the system can be fully described by two equations of state.

For strain-rate gradient values $\dot{e} \le \dot{e}_1$, the rheological behavior of the system is described by a quasi-exponential law

$$\tau = \tau_0 + k\dot{e}\,|\dot{e}^{n-1}| \quad (\tau_0 \geqslant 0, \quad n < 1). \tag{1}$$

With values of $\dot{e} \ge \dot{e}_1$, the rheological curve changes into a region of viscoplastic flow which is described by the equation

$$\tau = \tau_0' + \eta\dot{e}. \tag{2}$$

Rheological flow curves that were similar in nature were also attained by Rebinder [1], Mirzadzhanzade [2], et al. Volarovich and Gutkin [3, 4] dealt with the motion of a viscoplastic system in cylinders.

First we solve the problem of the motion (in a cylindrical tube) of a system which, in various segments of the cross section, is described by a variety of rheological equations. This is a new approach to the solution of the problem and makes it possible to derive a simpler solution for the cases in which the system experiences both linear and angular strain, and where the solution of the differential equations of motion is carried through to the integration of complex nonlinear equations [5]. For example, with motion in a cone through a region subject to a quasi-exponential law, we can neglect the angular deformations which are small in this case relative to the linear strains. However, in the viscoplastic flow region near the walls, conversely, the angular strains exceed the linear deformations substantially and the latter can now be neglected in the solution of the problem. This simplifies the solution significantly and, as we can see from the problem of motion in a circular cylindrical tube (Fig. 2), the curve showing the velocity distribution in this event is in better agreement than those derived experimentally.

Having written Eq. (1) in tensor form, and solving it simultaneously with the Cauchy equilibrium equation, we obtain a general mechanics equation for a quasi-exponential medium which, in orthogonal curvilinear coordinates, assumes the form

$$\left[-2\frac{\tau_0}{h^2} + 2k(n-1)h^{n-2}\right]\sum_{i=1}^{3}\frac{1}{H_i}\frac{dh}{dx^i}\,\dot{e}_{ik} + 2\left(\frac{\tau_0}{h} + kh^{n-1}\right)\frac{1}{H_k}\times$$
$$\times\sum_{\beta=1}^{3}\left[\frac{1}{H_1H_2H_3}\frac{\partial\left(\frac{H_1H_2H_3H_k}{H_\beta}\dot{e}_{k\beta}\right)}{\partial x^\beta} - \dot{e}_{\beta\beta}\frac{\partial \ln H_\beta}{\partial x^k}\right] - \frac{1}{H_k}\frac{\partial p}{\partial x^k} = \rho_0\left[\frac{\partial v_{x^k}}{\partial t} + \sum_{\beta=1}^{3}\frac{v_{x^\beta}}{H_\beta}\times\right.$$
$$\left.\times\left(\frac{\partial v_{x^k}}{\partial x^\beta} + \frac{v_{x^k}}{H_k}\frac{\partial H_k}{\partial x^\beta} - \frac{v_{x^\beta}}{H_k}\frac{\partial H_\beta}{\partial x^k}\right)\right]. \tag{3}$$

For the viscoplastic flow zone

$$2\left(\eta + \frac{\tau_0}{h}\right)\frac{1}{H_k}\times\sum_{\beta=1}^{3}\left[\frac{1}{H_1H_2H_3}\frac{\partial\left(\frac{H_1H_2H_3H_k}{H_\beta}\dot{e}_{k\beta}\right)}{\partial x^\beta} - \dot{e}_{\beta\beta}\frac{\partial \ln H_\beta}{\partial x^k}\right] - 2\frac{\tau_0}{h^2}\sum_{\beta=1}^{3}\frac{1}{H_\beta}\frac{\partial h}{\partial x^\beta}\dot{e}_{k\beta} -$$
$$-\frac{1}{H_k}\frac{\partial p}{\partial x^k} = \rho_0\left[\frac{\partial v_{x^k}}{\partial t} + \sum_{\beta=1}^{3}\frac{v_{x^\beta}}{H_\beta}\times\left(\frac{\partial v_{x^k}}{\partial x^\beta} + \frac{v_{x^k}}{H_k}\frac{\partial H_k}{\partial x^\beta} - \frac{v_{x^\beta}}{H_k}\frac{\partial H_\beta}{\partial x^k}\right)\right]. \tag{4}$$

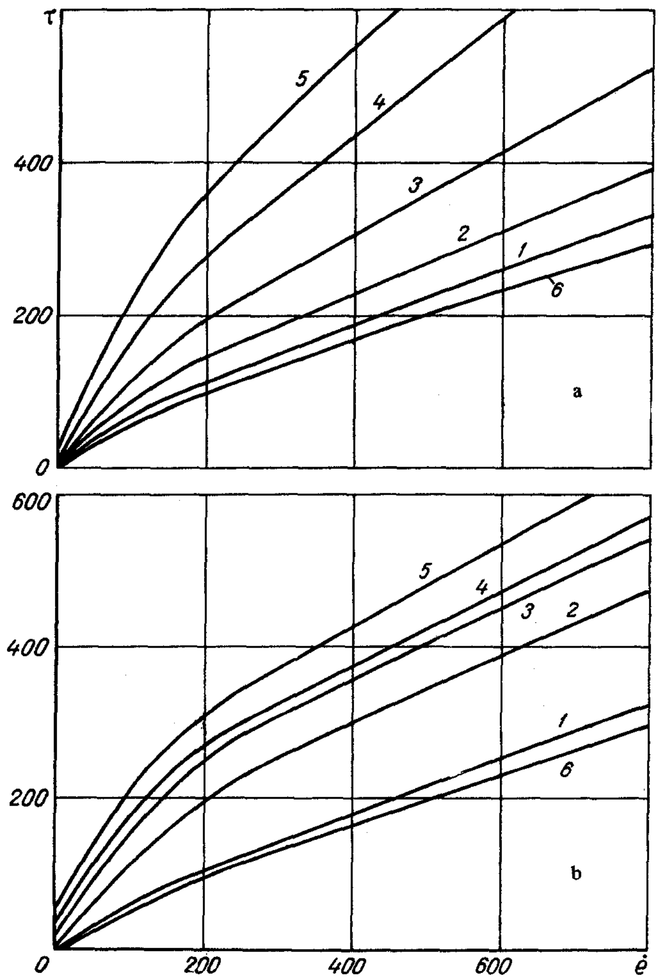

Fig. 1. Rheological flow curves of polymer solutions with fillings (τ, N/m^2; e, sec^{-1}); a) 10 per cent solution of chlorosulfur polyethylene in xylene filled with cement; b) solution of polystyrene in xylene filled with pigments: 1) 16.67 per cent filler, 2) 28.58, 3) 37.5, 4) 44.4, 5) 50 per cent, 6) without filler.

In solving the problem of motion in a circular cylindrical tube, we proceed from the fact that the streamlines are straight and parallel to the cylindrical axis. In the cylindrical coordinate system for an axisymmetric flow $v_r = 0$, $v_\varphi = 0$, and $v_z = f(r)$. The components of the strainrate tensor are thus the following:

$$\dot e_{rr} = 0;\quad \dot e_{\varphi\varphi} = 0;\quad \dot e_{zz} = 0;\quad \dot e_{r\varphi} = 0;$$

$$\dot e_{z\varphi} = 0;\quad \dot e_{rz} = \frac{1}{2}\,\frac{\partial v_z}{\partial r}.$$

The continuity equation for the strain rates is thus satisfied identically. On this basis, we obtain

$$\frac{\partial p}{\partial z} = -\alpha;$$

$$\left[-2\frac{\tau_0}{h^2} + 2k(n-1)h^{n-2}\right]\frac{\partial h}{\partial r}\,\dot e_{rz} + 2\left(\frac{\tau_0}{h} + kh^{n-1}\right)\frac{1}{r}\,\frac{\partial (r\dot e_{rz})}{\partial r} + \alpha = 0. \qquad (5)$$

Solution of Eq. (5) yields

$$v_z = v_0 - \frac{2kn}{\alpha(n+1)}\left(\frac{\alpha r}{2k} - \frac{\tau_0}{k}\right)^{\frac{1}{n}+1}. \qquad (6)$$

The integration constants are found from the following conditions: C_1 is found from the condition of equilibrium for the flow core; C_2 is found from the condition that when $r = r_0$, $v_z = v_0 = v_{max}$.

Equation (6) corresponds to the velocity distribution in the region of quasi-exponential flow for values of $r \le r_1$. When $r_1 \le r \le R$, the system moves in the region of viscoplastic flow. Having written Eq. (4) in projections onto the axis of the cylindrical coordinate system, we obtain the following by solving the expression for the velocity in this zone:

$$v_{z'} = \frac{\tau_{0'}}{\eta}(r - R) + \frac{\alpha}{4\eta}(R^2 - r^2), \qquad (7)$$

$C_1 = 0$, and C_2 is found from the condition of the adhesion of the system to the nonmoving wall.

The condition $2\pi r_1 l \tau_1 = p\pi r_1^2$ yields

$$r_1 = \frac{2\tau_1}{\alpha}. \qquad (8)$$

Here τ_1 is the magnitude of the tangential shear stress which corresponds to the transition of the motion from a quasi-exponential regime into the region of viscoplastic flow, and this quantity is determined experimentally.

The condition that $r = r_1$ when $v_z = v_z'$ gives us the value of the maximum velocity

$$v_0 = \frac{2kn}{\alpha(n+1)}\left(\frac{\tau_1}{k} - \frac{\tau_0}{k}\right)^{\frac{1}{n}+1} + \frac{\tau_{0'}}{\eta}\times \left(\frac{2\tau_1}{\alpha} - R\right) + \frac{\alpha}{4\eta}\left(R^2 - \frac{4\tau_1}{\alpha^2}\right). \qquad (9)$$

We can determine the relationship between pressure and flow rate from the condition of flow-rate constancy:

$$Q = \pi r_0^2 v_0 + 2\pi\int_{r_0}^{r_1} v_z r\,dr + 2\pi\int_{r_1}^{R} v_{z'} r\,dr. \qquad (10)$$

The results of the solution show that when $\tau_0 \ne 0$, there will be three zones with various velocity-distribution functions within the region of motion. In the

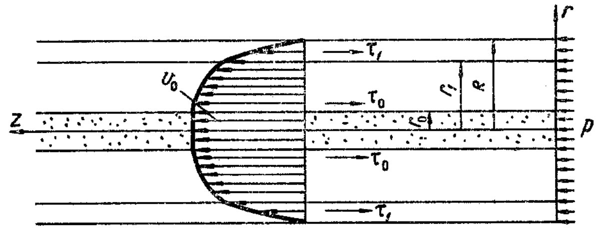

Fig. 2. Plot of velocity distribution in circular cylindrical tube.

first zone—about the axis—we have $\tau \le \tau_0$. This zone moves in the manner of a solid and all of its points exhibit identical velocity. The relationship between the tangential stress and the strain rate is expressed for the second zone by Eq. (1). Here we have a velocity gradient, but it is small in absolute magnitude. In the third zone (the Bingham flow zone) the relationship between the tangential stress and the strain rate is expressed by Eq. (2). We will find the greatest tangential stresses in this zone.

NOTATION

$\dot e$ is the gradient of the strain rate; τ is the tangential shear stress; τ_0 is the limit shear stress; η is the plastic viscosity; k and n are the rheological constants; h is the intensity of strain rate; p is the hydrostatic pressure, α is the piezometric slope; H_k is the scale factor; r is the radial coordinate; r_0 is the radius of the flow core; R is the tube radius; ρ is the density; l is the tube length.

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31 August 1967

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